Comment on "Failure of the Jarzynski identity for a simple quantum system."

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The article of Engel and Nolte [1] is based on an incorrect definition of work. In fact, the Jarzynski identity does hold. The energy and work distribution of a driven, but otherwise isolated, quantum system may be calculated using a time-dependent (adiabatic) basis which diagonalizes the Hamiltonian at a given time. The general proof of the Jarzynski relation for an arbitrary driving protocol was given in ref. [2]. Let us consider the special case discussed in [1] where we suddenly switch the Hamiltonian H_0 into H_1. We denote the corresponding partition functions Z_0 and Z_1. Consider a quantum trajectory where the system starts in an eigenstate φ_j of H_0 with eigenvalue ε_j, and ends up in an eigenstate Ψ_n of H_1 with eigenvalue ε_n. The probability of this trajectory is

$$P_{nj} = \frac{\exp(-\beta\varepsilon_j)}{Z_0}|c_{nj}|^2, \tag{0.1}$$

where c_{nj} are the expansion coefficients of φ_j in the new basis $\varphi_j = \sum_n c_{nj}\Psi_n$, with $\sum_j |c_{nj}|^2$ = 1. Since there is no bath, the work done on the system is $W_{nj} = \varepsilon_n$ - ε_j. Using these definitions we immediately recover the Jarzynski relation

$$\langle e^{-\beta W}\rangle \equiv \sum_{jn} e^{-\beta W_{nj}} P_{nj} = \frac{Z_1}{Z_0} \equiv \exp(-\beta\Delta F) \tag{0.2}$$

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[1] Engel, A., Nolte, R. cond-mat/0612527v1 (2006)

[2] Mukamel, S. Phys. Rev. Lett. 90, 170604 (2003)